

Surface symmetries and surface point groups for representative faces of the 32 point groups

For each point group, symmetry elements are listed. The table indicates the symmetry elements conserved at the surface of each face. Only faces representative of the crystal form are listed.

At the end of each column, the final point group of the surface (i.e. *surface point group*) is indicated in bold.

Nomenclature used for symmetry elements :

For symmetry axes ($S = 2, 3, 4, 6, \bar{2}, \bar{3}, \bar{4}, \bar{6}$): the direction of the axis is indicated.

For mirrors ($S = m$), the normal to the plane is indicated.

Examples: $2[001]$ is a twofold axis running along the $[001]$ direction, and $m[010]$ is a mirror perpendicular to the y axis.

Corresponding transformation matrices are available online on the Bilbao Crystallographic Server: <http://www.cryst.ehu.es/rep/point.html>

1 Trigonal system

Point group 1

Symmetry elements: 1

$\{100\}$	$\{010\}$	$\{001\}$	$\{0kl\}$	$\{h0l\}$	$\{hk0\}$	$\{hkl\}$
1	1	1	1	1	1	1
1	1	1	1	1	1	1

Point group $\bar{1}$

Symmetry elements: 1, $\bar{1}$

$\{100\}$	$\{010\}$	$\{001\}$	$\{0kl\}$	$\{h0l\}$	$\{hk0\}$	$\{hkl\}$
1	1	1	1	1	1	1
1	1	1	1	1	1	1

2 Monoclinic system

Point group 2

Symmetry elements: 1, 2[010]

{100}	{010}	{001}	{0kl}	{h0l}	{hk0}	{hkl}
1	1	1	1	1	1	1
	2[010]					
1	2	1	1	1	1	1

Point group *m*

Symmetry elements: 1, m[010]

{100}	{010}	{001}	{0kl}	{h0l}	{hk0}	{hkl}
1	1	1	1	1	1	1
m[010]		m[010]		m[010]		
m	1	m	1	m	1	1

Point group 2/*m*

Symmetry elements: 1, $\bar{1}$, 2[010], m[010]

{100}	{010}	{001}	{0kl}	{h0l}	{hk0}	{hkl}
1	1	1	1	1	1	1
m[010]	2[010]	m[010]		m[010]		
m	2	m	1	m	1	1

3 Orthogonal system

Point group 222

Symmetry elements: 1, 2[100], 2[010], 2[001]

{100}	{010}	{001}	{0kl}	{h0l}	{hk0}	{hkl}
1	1	1	1	1	1	1
2[100]	2[010]	2[001]				
2	2	2	1	1	1	1

Point group *mm2*

Symmetry elements: 1, m[100], m[010], m[001]

$\{100\}$	$\{010\}$	$\{001\}$	$\{0kl\}$	$\{h0l\}$	$\{hk0\}$	$\{hkl\}$
1	1	1	1	1	1	1
m[010]	m[100]	m[100] m[010] 2[001]	m[100]	m[010]		
m	m	mm2	m	m	1	1

Point group mmm

Symmetry elements: 1, 2[100], 2[010], 2[001], $\bar{1}$, m[100], m[010], m[001]

$\{100\}$	$\{010\}$	$\{001\}$	$\{0kl\}$	$\{h0l\}$	$\{hk0\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[100]	2[010]	2[001]	m[100]	m[010]	m[001]	
m[001]	m[001]	m[010]				
m[010]	m[100]	m[100]				
2mm	m2m	mm2	m	m	m	1

4 Tetragonal system

Point group 4

Symmetry elements: 1, 2[001], 4¹[001], 4³[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]						
4 ¹ [001]						
4 ³ [001]						
4	1	1	1	1	1	1

Point group $\bar{4}$

Symmetry elements: 1, 2[001], $\bar{4}^1$ [001], $\bar{4}^3$ [001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]						
2	1	1	1	1	1	1

Point group $4/m$

Symmetry elements: 1, $\bar{1}$, 2[001], m[001], 4¹[001], 4³[001], $\bar{4}^1$ [001], $\bar{4}^3$ [001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]	m[001]	m[001]	m[001]			
4 ¹ [001]						
4 ³ [001]						
4	m	m	m	1	1	1

Point group 422

Symmetry elements: 1, 2[100], 2[010], 2[001], 2[110], 2[1 $\bar{1}$ 0], 4¹[001], 4³[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]	2[100]	2[110]				
4 ¹ [001]						
4 ³ [001]						
4	2	2	1	1	1	1

Point group 4mm

Symmetry elements: 1, 2[001], 4¹[001], 4³[001], m[100], m[010], m[110], m[1 $\bar{1}$ 0]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]	m[010]	m[1 $\bar{1}$ 0]		m[010]	m[1 $\bar{1}$ 0]	
4 ¹ [001]						
4 ³ [001]						
m[100]						
m[010]						
m[110]						
m[1 $\bar{1}$ 0]						
4	m	m	1	m	m	1

Point group $\bar{4}2m$

Symmetry elements: 1, 2[100], 2[010], 2[001], m[110], m[1 $\bar{1}$ 0], $\bar{4}$ ¹[001], $\bar{4}$ ³[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]	2[100]	m[1 $\bar{1}$ 0]			m[1 $\bar{1}$ 0]	
m[110]						
m[1 $\bar{1}$ 0]						
mm2	2	m	1	1	m	1

Point group 4/mmm

Symmetry elements: 1, 2[100], 2[010], 2[001], 2[110], 2[1 $\bar{1}$ 0], 4¹[001], 4³[001], $\bar{1}$, m[100], m[010], m[001], m[110], m[1 $\bar{1}$ 0], $\bar{4}$ ¹[001], $\bar{4}$ ³[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]	2[100]	2[110]	m[001]	m[010]	m[1 $\bar{1}$ 0]	
4 ¹ [001]	m[001]	m[001]				
4 ³ [001]	m[010]	m[1 $\bar{1}$ 0]				
m[010]						
m[100]						
m[110]						
m[1 $\bar{1}$ 0]						
4mm	2mm	2mm	m	m	m	1

5 Trigonal system

Point group 3

Symmetry elements: 1, 3¹[001], 3²[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
3 ¹ [001]						
3 ² [001]						
3	1	1	1	1	1	1

Point group $\bar{3}$

Symmetry elements: 1, 3¹[001], 3²[001], $\bar{1}$, $\bar{3}^1$ [001], $\bar{3}^2$ [001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
3 ¹ [001]						
3 ² [001]						
3	1	1	1	1	1	1

Point group 32

Symmetry elements: 1, 2[1 $\bar{1}$ 0], 2[210], 2[120], 3¹[001], 3²[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
3 ¹ [001]	2[210]					
3 ² [001]						
3	2	1	1	1	1	1

Point group $3m$

Symmetry elements: 1, $3^1[001]$, $3^2[001]$, $m[1\bar{1}0]$, $m[210]$, $m[120]$

{001}	{100}	{110}	{hk0}	{h0l}	{hhl}	{hkl}
1	1	1	1	1	1	1
$3^1[001]$		$m[1\bar{1}0]$			$m[1\bar{1}0]$	
$3^2[001]$						
$m[1\bar{1}0]$						
$m[210]$						
$m[120]$						
3m	1	m	1	1	m	1

Point group $\bar{3}m$

Symmetry elements: 1, $2[1\bar{1}0]$, $2[210]$, $2[120]$, $3^1[001]$, $3^2[001]$, $\bar{1}$, $m[1\bar{1}0]$, $m[210]$, $m[120]$, $\bar{3}^1[001]$, $\bar{3}^2[001]$

{001}	{100}	{110}	{hk0}	{h0l}	{hhl}	{hkl}
1	1	1	1	1	1	1
$3^1[001]$	$2[210]$	$m[1\bar{1}0]$			$m[1\bar{1}0]$	
$3^2[001]$						
$m[1\bar{1}0]$						
$m[210]$						
$m[120]$						
3m	2	m	1	1	m	1

6 Hexagonal system

Point group 6

Symmetry elements: 1, $2[001]$, $3^1[001]$, $3^2[001]$, $6^1[001]$, $6^5[001]$

{001}	{100}	{110}	{hk0}	{h0l}	{hhl}	{hkl}
1	1	1	1	1	1	1
$2[001]$						
$3^1[001]$						
$3^2[001]$						
$6^1[001]$						
$6^5[001]$						
6	1	1	1	1	1	1

Point group $\bar{6}$

Symmetry elements: 1, $3^1[001]$, $3^2[001]$, $m[001]$, $\bar{6}^1[001]$, $\bar{6}^5[001]$

{001}	{100}	{110}	{hk0}	{h0l}	{hhl}	{hkl}
1	1	1	1	1	1	1
$3^1[001]$	$m[001]$	$m[001]$	$m[001]$			
$3^2[001]$						
3	m	m	m	1	1	1

Point group $6/m$

Symmetry elements: 1, $2[001]$, $3^1[001]$, $3^2[001]$, $6^1[001]$, $6^5[001]$, $\bar{1}$, $m[001]$, $\bar{3}^1[001]$, $\bar{3}^2[001]$, $\bar{6}^1[001]$, $\bar{6}^5[001]$

{001}	{100}	{110}	{hk0}	{h0l}	{hhl}	{hkl}
1	1	1	1	1	1	1
$2[001]$	$m[001]$	$m[001]$	$m[001]$			
$3^1[001]$						
$3^2[001]$						
$6^1[001]$						
$6^5[001]$						
6	m	m	m	1	1	1

Point group 622

Symmetry elements: 1, $2[100]$, $2[0\bar{1}0]$, $2[001]$, $2[110]$, $2[1\bar{1}0]$, $2[210]$, $2[120]$, $3^1[001]$, $3^2[001]$, $6^1[001]$, $6^5[001]$

{001}	{100}	{110}	{hk0}	{h0l}	{hhl}	{hkl}
1	1	1	1	1	1	1
$2[001]$	$2[210]$	$2[110]$				
$3^1[001]$						
$3^2[001]$						
$6^1[001]$						
$6^5[001]$						
6	2	2	1	1	1	1

Point group $6mm$

Symmetry elements: 1, $2[001]$, $3^1[001]$, $3^2[001]$, $6^1[001]$, $6^5[001]$, $m[100]$, $m[0\bar{1}0]$, $m[110]$, $m[1\bar{1}0]$, $m[210]$, $m[120]$

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]	m[0 $\bar{1}$ 0]	m[1 $\bar{1}$ 0]		m[0 $\bar{1}$ 0]	m[1 $\bar{1}$ 0]	
3 ¹ [001]						
3 ² [001]						
6 ¹ [001]						
6 ⁵ [001]						
m[100]						
m[0 $\bar{1}$ 0]						
m[110]						
m[1 $\bar{1}$ 0]						
m[210]						
m[120]						
6mm	m	m	1	m	m	1

Point group $\bar{6}m2$

Symmetry elements: 1, 2[100], 2[0 $\bar{1}$ 0], 2[110], 3¹[001], 3²[001], m[001], m[1 $\bar{1}$ 0], m[210], m[120], $\bar{6}$ ¹[001], $\bar{6}$ ⁵[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
3 ¹ [001]	m[001]	m[001]	m[001]		m[1 $\bar{1}$ 0]	
3 ² [001]		2[110]				
m[1 $\bar{1}$ 0]		m[1 $\bar{1}$ 0]				
m[210]						
m[120]						
3m	m	2mm	m	1	m	1

Point group $6/mmm$

Symmetry elements: 1, 2[100], 2[0 $\bar{1}$ 0], 2[001], 2[110], 2[1 $\bar{1}$ 0], 2[210], 2[120], 3¹[001], 3²[001], 6¹[001], 6⁵[001], $\bar{1}$, m[100], m[0 $\bar{1}$ 0], m[001], m[110], m[1 $\bar{1}$ 0], m[210], m[120], $\bar{3}$ ¹[001], $\bar{3}$ ²[001], $\bar{6}$ ¹[001], $\bar{6}$ ⁵[001]

$\{001\}$	$\{100\}$	$\{110\}$	$\{hk0\}$	$\{h0l\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1	1
2[001]	2[210]	2[110]	m[001]	m[0 $\bar{1}$ 0]	m[1 $\bar{1}$ 0]	
3 ¹ [001]	m[001]	m[001]				
3 ² [001]	m[0 $\bar{1}$ 0]	m[1 $\bar{1}$ 0]				
6 ¹ [001]						
6 ⁵ [001]						
m[100]						
m[0 $\bar{1}$ 0]						
m[110]						
m[1 $\bar{1}$ 0]						
m[210]						
m[120]						
6mm	2mm	2mm	m	m	m	1

7 Cubic system

Point group 23

Symmetry elements: 1, 2[100], 2[010], 2[001], 3¹[111], 3²[111], 3¹[1 $\bar{1}$ 1], 3²[1 $\bar{1}$ 1], 3¹[11 $\bar{1}$], 3²[11 $\bar{1}$], 3¹[1 $\bar{1}$ $\bar{1}$], 3²[1 $\bar{1}$ $\bar{1}$]

$\{001\}$	$\{111\}$	$\{011\}$	$\{0kl\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1
2[001]	3 ¹ [111] 3 ² [111]				
2	3	1	1	1	1

Point group $m\bar{3}$

Symmetry elements: 1, 2[100], 2[010], 2[001], 3¹[111], 3²[111], 3¹[1 $\bar{1}$ 1], 3²[1 $\bar{1}$ 1], 3¹[11 $\bar{1}$], 3²[11 $\bar{1}$], 3¹[1 $\bar{1}$ $\bar{1}$], 3²[1 $\bar{1}$ $\bar{1}$], $\bar{1}$, m[100], m[010], m[001], $\bar{3}$ ¹[111], $\bar{3}$ ²[111], $\bar{3}$ ¹[1 $\bar{1}$ 1], $\bar{3}$ ²[1 $\bar{1}$ 1], $\bar{3}$ ¹[11 $\bar{1}$], $\bar{3}$ ²[11 $\bar{1}$], $\bar{3}$ ¹[1 $\bar{1}$ $\bar{1}$], $\bar{3}$ ²[1 $\bar{1}$ $\bar{1}$]

$\{001\}$	$\{111\}$	$\{011\}$	$\{0kl\}$	$\{hhl\}$	$\{hkl\}$
1	1	1	1	1	1
2[001]	3 ¹ [111]	m[100]	m[100]		
m[010]	3 ² [111]				
m[100]					
2mm	3	m	m	1	1

Point group 432

Symmetry elements: 1, 2[100], 2[010], 2[001], 2[110], 2[1 $\bar{1}$ 0], 2[011], 2[01 $\bar{1}$], 2[101], 2[10 $\bar{1}$], 3¹[111], 3²[111], 3¹[1 $\bar{1}$ 1], 3²[1 $\bar{1}$ 1], 3¹[11 $\bar{1}$], 3²[11 $\bar{1}$], 3¹[1 $\bar{1}$ $\bar{1}$], 3²[1 $\bar{1}$ $\bar{1}$], 4¹[100], 4³[100], 4¹[010], 4³[010], 4¹[001], 4³[001]

{001}	{111}	{011}	{0kl}	{hhl}	{hkl}
1	1	1	1	1	1
2[001]	3 ¹ [111]	2[011]			
4 ¹ [001]	3 ² [111]				
4 ³ [001]					
4	3	2	1	1	1

Point group $\bar{4}3m$

Symmetry elements: 1, 2[100], 2[010], 2[001], 3¹[111], 3²[111], 3¹[1 $\bar{1}$ 1], 3²[1 $\bar{1}$ 1], 3¹[11 $\bar{1}$], 3²[11 $\bar{1}$], 3¹[1 $\bar{1}$ $\bar{1}$], 3²[1 $\bar{1}$ $\bar{1}$], m[110], m[1 $\bar{1}$ 0], m[011], m[01 $\bar{1}$], m[101], m[10 $\bar{1}$], 4¹[100], 4³[100], 4¹[010], 4³[010], 4¹[001], 4³[001]

{001}	{111}	{011}	{0kl}	{hhl}	{hkl}
1	1	1	1	1	1
2[001]	3 ¹ [111]	m[01 $\bar{1}$]		m[1 $\bar{1}$ 0]	
m[110]	3 ² [111]				
m[1 $\bar{1}$ 0]	m[1 $\bar{1}$ 0]				
	m[01 $\bar{1}$]				
	m[10 $\bar{1}$]				
2mm	3m	m	1	m	1

Point group $m\bar{3}m$

Symmetry elements: 1, 2[100], 2[010], 2[001], 2[110], 2[1 $\bar{1}$ 0], 2[011], 2[01 $\bar{1}$], 2[101], 2[10 $\bar{1}$], 3¹[111], 3²[111], 3¹[1 $\bar{1}$ 1], 3²[1 $\bar{1}$ 1], 3¹[11 $\bar{1}$], 3²[11 $\bar{1}$], 3¹[1 $\bar{1}$ $\bar{1}$], 3²[1 $\bar{1}$ $\bar{1}$], 4¹[100], 4³[100], 4¹[010], 4³[010], 4¹[001], 4³[001], $\bar{4}$, m[100], m[010], m[001], m[110], m[1 $\bar{1}$ 0], m[011], m[01 $\bar{1}$], m[101], m[10 $\bar{1}$], 3¹[111], 3²[111], 3¹[1 $\bar{1}$ 1], 3²[1 $\bar{1}$ 1], 3¹[11 $\bar{1}$], 3²[11 $\bar{1}$], 3¹[1 $\bar{1}$ $\bar{1}$], 3²[1 $\bar{1}$ $\bar{1}$], 4¹[100], 4³[100], 4¹[010], 4³[010], 4¹[001], 4³[001]

{001}	{111}	{011}	{0kl}	{hhl}	{hkl}
1	1	1	1	1	1
2[001]	3 ¹ [111]	2[011]	m[100]	m[1 $\bar{1}$ 0]	
4 ¹ [001]	3 ² [111]	m[100]			
4 ³ [001]	m[1 $\bar{1}$ 0]	m[01 $\bar{1}$]			
m[100]	m[01 $\bar{1}$]				
m[010]	m[10 $\bar{1}$]				
m[110]					
m[1 $\bar{1}$ 0]					
4mm	3m	2mm	m	m	1